



## Optimization of Neuron Models Using Advanced Algorithms

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### ABSTRACT

Conductance based model of biological neuron can accurately reproduce the waveform of the membrane voltage as well as the spike timing in response to injected currents. Some of the remarkable models are Hodgkin Huxley (H-H) conductance based neuron model, Fitzhugh model and IZ model. These models have different elements and parameters, the values of which determine the trajectory of the membrane voltage. It is, therefore, important to find the optimized parameter values for proper reproduction of the action potential of neuron. Here we have used three algorithms namely, Genetic Algorithm (GA), Particle swarm optimization (PSO) and Firefly Algorithm (FA) to estimate the parameters of three neuron models and compared them with a reference signal. It is found that FA is more accurate and efficient in terms of estimation parameters of neuron model.

**Keywords:** Hodgkin-Huxley model, Fitzhugh model, Izekevich model, Firefly Algorithm.

### INTRODUCTION

There are various conductance based mathematical models developed so far<sup>1-18</sup>. These models can reproduce the various neuron signals associated with it. To produce the required signals, it is necessary to have a good estimate of the parameters associated with it. Although there are numerous methods for estimating the parameters, here genetic algorithm, particle swarm optimization and firefly method are taken into consideration.

These methods were chosen due to the fact that they are based on evolutionary algorithms; nature inspired and is advanced optimization methods so far. These methods are easy to use and efficient. The neuron models considered in this paper are the infamous H-H model, IZ and Fitzhugh models. These neuron models parameters are compared by the optimization methods with the actual values obtained by the models and their accuracy is observed. Estimation of parameters is an important field in neurology since it will help to find out the abnormality in a signal and the reason behind it.

For estimating the parameters, an algorithm is required and a fitness function which will reproduce the signal. Here, particle swarm optimization (PSO) method, genetic algorithm (GA) and firefly algorithm (FA) is used for estimating the parameters. PSO is an advanced optimization algorithm where it follows the principle of searching food by bird flocks<sup>2-7</sup>. It constantly searches for better solution obtained by neighboring particle. The solution then converges towards the best solution obtained so far. Firefly optimization method is a recent developed optimization method. It is a meta heuristic method inspired from nature. It uses the principle of fireflies who attracts the other fireflies by its brightness. Genetic algorithm is an evolutionary algorithm. It uses the

principle of Darwin's survival of the fittest from the random population generated. The fittest ones are selected for mating, crossover and mutation<sup>8-14</sup>. This process goes on until the best solution is found. These three methods were applied to the mathematical models of neuron to extract the essential parameters. The three methods were compared on account of their reliability, accuracy, speed and efficiency. The different models which are used for estimating the parameters are described in the sections below.

### Hodgkin-Huxley model

The extracellular fluid and the inside of the cell is separated by a semi permeable membrane which acts as a capacitor. This ionic charge is disturbed when an external stimulus is exerted on it and adds further charge. The ionic concentration is different in either sides of the membrane and it is called Nernst potential and is represented by  $E_{Na}$ ,  $E_K$ , and  $E_I$  for sodium, potassium and leakage ions respectively. The total current flow is divided into capacitive current, sodium ions, potassium ions and leakage ions as shown in equation (1) and (2).

$$I = C_{membrane} \frac{dV}{dt} + I_{ion} \quad (1)$$

$$I = C_{membrane} \frac{dV}{dt} + g_K n^4 (V - V_K) + g_{Na} m^3 h (V - V_{Na}) + g_I (V - V_I) \quad (2)$$

Each individual current is represented by its conductance and difference between the membrane potential (V) and equilibrium potential of each ions ( $V_{Na}$ ,  $V_K$ ,  $V_I$ ). m, n, h are the probability of opening of the channels of sodium and potassium respectively. They are generally referred as gating variables; m and h are the probability of activation and inactivation of the sodium channels. n is the probability of activation of potassium ions.  $\alpha_m$ ,  $\beta_m$ ,  $\beta_h$  are



the rate constants which are responsible for flow of ions from outside to inside and vice versa for  $\beta_m, \alpha_h, \beta_n$  of sodium ions and potassium ions respectively<sup>1-4</sup>. These equations were found by Hodgkin-Huxley by curve fitting methods. The above variables will be estimated by using genetic algorithm, particle swarm optimization method, firefly optimization method.

**Fitghugh model**

Fitghugh model is a reduced form of H-H model. It is a mathematical model describing the neuron. It takes the principle of excitation and propagation of action potential. The mathematical description of the model takes the form shown in equation (3) and (4). It is observed from H-H equation that n and h are slow and summed up to a very small value. Fitghugh have reduced the neuron model therefore in two dimensional models. z represents the external current stimulation in a neuron, x is the membrane potential. y is the recovery variable necessary to attain resting potential<sup>5</sup>.

$$\frac{dx}{dt} = c(x - y + z - \frac{x^3}{3}) \tag{3}$$

$$\frac{dy}{dt} = (x - by + a) / c \tag{4}$$

**Izhevich model**

Izhikevich have mathematically given the following equations<sup>18</sup>:

$$\dot{v} = 0.04v^2 + 5v + 140 - u + I \tag{5}$$

$$u' = a(bv - u) \tag{6}$$

If  $v = 30$  mV then  $v \leftarrow c, u \leftarrow u + d$

a, b, c, d are the constant parameter and are dimensionless. v is the membrane voltage and u is the recovery variable and it is constantly changed to get the required signal. a is the timing variable recovery, b determines the sensitivity of the recovery variable and the oscillations of the spikes to v, c is the recovery variable for v due to potassium ions after the spike and d represents reset of variable u due to conductance of sodium and potassium. The two simple equation can produce different type of spikes by varying the variables unlike H-H model which takes into account many differential equations to produce the action potential. By changing the values of a, b, c, d one can generate different types of spikes such as regular spikes, fast spikes, slow threshold spikes etc.

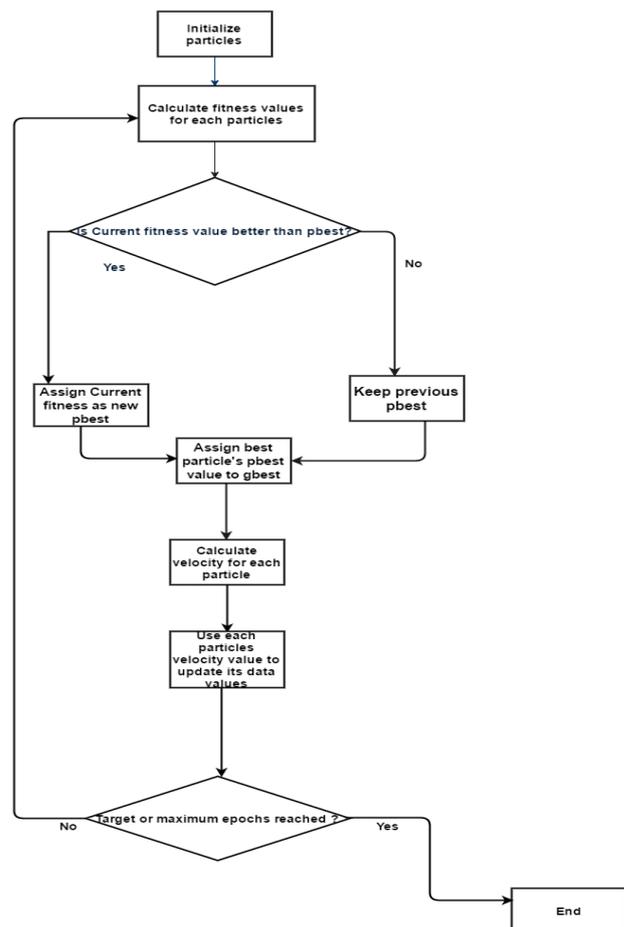
**MATERIALS AND METHODS**

Estimation of parameters in the above models requires a good optimization method. Genetic Algorithm (GA), Firefly algorithm (FA) optimization method and particle swarm optimization method (PSO) are used here. For any estimation method, fitness function is required. Fitness function is obtained by the equations given in the section

above. A reference signal is also needed to compare the signals and for exact and accurate estimation of the parameters. Reference signals are taken from the signals given by Hodgkin-Huxley, IZ, and Fitghugh<sup>15-19</sup>. The optimization methods were coded in MATLAB 2010a software for estimation purpose.

The parameters described in the equations in the section above are estimated. The algorithms were applied in MATLAB to estimate the parameters related to the neuron signals<sup>12-19</sup>.

GA is a meta heuristic method which searches for an optimum solution and follows the principle of evolution. A fitness function and its variables is defined which is optimized for better results in GA. GA uses the principle of Darwin's theory of natural selection. It generates a population using the variables and this population (parent) is used to generate the next generation (off springs). This process continues until a convergent to solution is found. While a new population is generated, two parents are chosen for better results. The parent is chosen according to the best result yield which is called selection process. In the next step, crossover is applied to form new off springs where the off springs contain the elements of both the parents equally. Mutation occurs which leads to sudden change and new population is replaced by the existing ones. This process is continued until the solution converges to best solution.



**Figure 1:** Flow Chart for PSO.

PSO is a meta heuristic algorithm (Fig.1) which follows the principle of flocking of birds established by Dr. Eberhart and Dr. Kennedy in 1995. Unlike GA, PSO does not follow the principle of evolution, crossover, mutation etc. In PSO, the solutions fly over to the optimum solution just like the flocking of birds.

Similar to birds who flock towards the bird who is closest to the food, the variables are kept closer to the solution desirable. Each particle in PSO searches for the best solution obtained so far called the pbest. The other particle then flocks towards the pbest solution. Another particle optimizer keeps track of the best solution obtained in the neighboring area called the lbest solution. gbest is the global best which tracks the best solution obtained taking into account all the population. This process is repeated until optimum solution is reached.

Third optimization method which is taken into account is the firefly algorithm. It is a nature inspired algorithm and found to be better than other algorithms like GA and PSO. Firefly algorithm follows the principle of fireflies following the brighter fireflies. Fireflies are attracted to other fireflies by their brightness level. The less bright ones are attracted to the brighter one and if it cannot find the brighter one than itself, the firefly moves randomly. The brightness  $I$  at a particular location  $x$  can be defined as  $I(x)$  is directly proportional to  $f(x)$ .  $\beta$  is the attractiveness factor and it is affected by distance  $r_{ij}$  where  $i$  and  $j$  are two different fireflies. The light intensity is defined by inverse square law given in equation (7):

$$I(r) = I_s / r^2 \tag{7}$$

Where  $I(s)$  is the intensity at the source.

In a fixed light absorption coefficient  $\gamma$ ,  $I$  (light intensity) depends with distance  $r$ .  $I_0$  is the original light intensity in equation (8).

$$I = I_0 e^{-\gamma r} \tag{8}$$

Both the above equation is combined to avoid  $r = 0$  and takes the form shown in equation (9):

$$I = I_0 e^{-\gamma r^2} \tag{9}$$

The distance between the fireflies  $i$  and  $j$  is the Cartesian distance:

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^d (x_{i,k} - x_{j,k})^2} \tag{10}$$

Where  $x_{ik}$  is the  $k^{\text{th}}$  component of  $x_i$ .

The movement of firefly  $i$  is attracted to another firefly  $j$  is explained as:

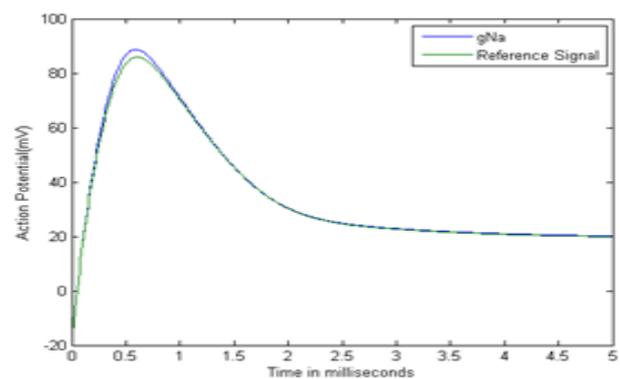
$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_j - x_i) + \alpha(\text{rand} - \frac{1}{2}) \tag{11}$$

Second term is the attraction factor and the third term is the randomization factor ( $\alpha$ ).  $\text{rand}$  is a random number

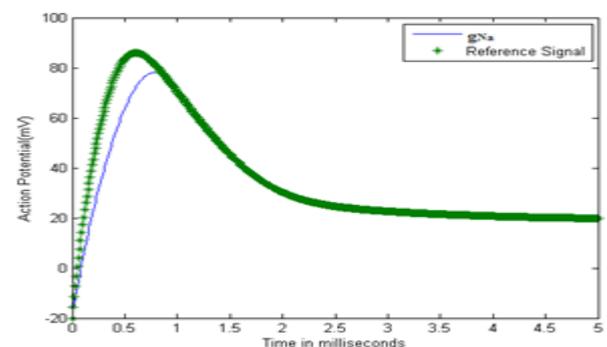
generated between 0 to 1. The above logic is used to implement the firefly algorithm.

**RESULTS AND DISCUSSION**

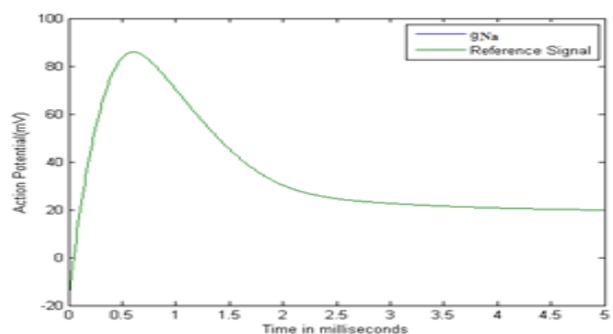
Figure 2 shows the action potential using GA in Hodgkin-Huxley model estimating the value of  $g_{Na}$ . Similarly, in Fig.3 and Fig.4 shows the action potential using PSO and FA with the reference signal estimating the value of  $g_{Na}$ . It is observed that value estimated by FA is more close to the value given by H-H model. In this way,  $g_k$  is estimated using these three optimization methods and shown in Fig.5, Fig. 6 Fig.7. It is seen that FA is more efficient than the other two methods. Using FA, the value is almost equal to the values given by H-H shown in Fig.7. Table 1(a) and Table 1(b) shows the theoretical and estimated values of the values of H-H model, IZ and Fitghugh using the three optimization methods.



**Figure 2:** Action Potential estimating the value of  $g_{Na}$  using GA in H-H model



**Figure 3:** Estimation of sodium conductance using PSO method



**Figure 4:** Estimation of sodium conductance in action potential using FA



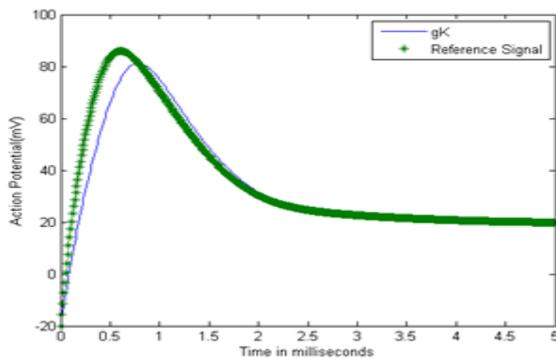
**Table 1(a):** Theoretical and estimated values of the values of H-H model.

Parameters	GA		PSO		FA	
	Theoretical	Estimated	Theoretical	Estimated	Theoretical	Estimated
$g_{\text{sodium}}$	120	200	120	180	120	120
$g_{\text{potassium}}$	36	20	36	22	36	36
$g_{\text{leakage}}$	0.3	0.2	0.3	0.2	0.3	0.3
$E_{\text{sodium}}$	-115	-110	-115	-111	-115	-115
$E_{\text{potassium}}$	12	10	12	11	12	12
$E_{\text{leakage}}$	-10.613	-10	-10.613	-10	-10.613	-10.6
$I$	0.1	0.3	0.1	0.1	0.1	0.09
$\alpha_m$	0.1	0.08	0.1	0.1	0.1	0.1
$\beta_m$	4	3.8	4	4	4	4
$\alpha_n$	0.01	0.01	0.01	0.02	0.01	0.01
$\beta_n$	0.125	0.2	0.125	0.2	0.125	0.125
$\alpha_h$	0.07	0.068	0.07	0.06	0.07	0.07
$\beta_h$	1	0.7	1	0.8	1	0.99

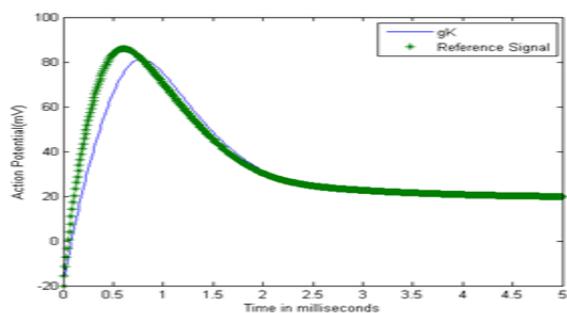
**Table 1(b):** Estimation of IZ and Fitghugh model using FA

Parameters	FA in Fitghugh model	
	Theoretical	Estimated
a	0.7	0.7
b	0.8	0.78
c	3	2.9
z	0.3	0.3

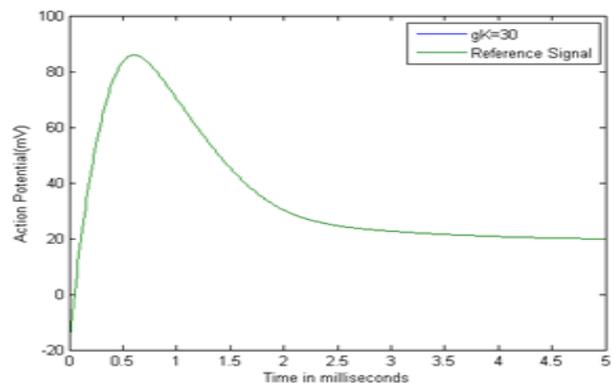
Parameters	FA in IZ model	
	Theoretical	Estimated
a	0.7	0.7
b	0.8	0.78
c	3	3
d	-16	-16
l	-99	-98.9



**Figure 5:** Estimation of potassium conductance using GA.



**Figure 6:** Estimation of potassium conductance using PSO method



**Figure 7:** Estimation of  $g_K$  using FA showing the convergence with the reference signal.

**CONCLUSION**

Estimation of various parameters in different neuron models are done here using three optimization methods. It has been found that FA is more accurate and efficient in terms of estimation. Speed can be increased by limiting the iterations in FA. Estimation of parameter will be a breakthrough in the area of neurology since the dependency of the parameters can be related. The biological signals of a patient can be also tested for any abnormality in the signal by parameter estimations. Many



other models and their parameters can also be estimated by this method.

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